The Definition and Computation of Modal Characteristic Impedance in Quasi-TEM Coupled Transmission Lines

G. G. Gentili, and M. Salazar-Palma, *Member, IEEE*

***Abstract*—The quasi-TEM analysis of systems of lossless coupled transmission lines in an inhomogeneous medium is reviewed. Starting from the generalized telegrapher's equations, the characteristic impedance of the normal modes is defined and computed according to the three usual definitions for the single-line case: power-current, power-voltage and voltage-current. Unlike the quasi-TEM single-line case, it is shown that the three definitions lead in general to different modal characteristic impedance values. Theoretical results are then confirmed by some numerical examples on two and three coupled-lines systems.**

I. Introduction

QUASI-TEM propagation in systems of coupled transmission lines in a inhomogeneous medium has been the subject of a great deal of work in the microwave and circuit area. Applications related to such structures range from the analysis and synthesis of classical microwave devices (directional couplers and parallel coupled resonators filters [1], [2]) to the interaction of complex structures through coupled transmission lines, such as high speed buses connecting logic circuits in modern digital computers [3]–[5].

Beside classical works [6], [7], which opened the way to rigorous time domain analysis of coupled transmission lines, some more recent literature has brought new contributions to the understanding of pulse propagation and distorsion both in the frame of high speed logic and of microwave field-oriented CAD tools [5], [8]–[10], [3], [4], [11]. Because of the growing speed and complexity of modern digital computers and microwave devices, a further impulse to the study of coupled structures in a inhomogeneous medium can be easily foreseen.

In the context of quasi-TEM coupled structures the methods for frequency and time-domain analysis are all based on the concept of normal modes [6]. In short, propagation in a system of N coupled transmission lines ($N + 1$ conductors) in an inhomogeneous medium can be described by N modes, which propagate decoupled from one another and with different speeds (in general).

Manuscript received October 21, 1993; revised April 14, 1994. This work was supported by the Spanish National Board of Scientific and Technological Research (CICYT) under project TIC93-0055-C03-01.

G. G. Gentili is with the "Centro Studi sulle Telecomunicazioni Spaziali" (CNR), Politecnico di Milano, Dipartimento di Elettronica e Informazione, 20133 Milano, Italy.

M. Salazar-Palma is with E.T.S.I. de Telecomunicacion, Universidad Politecnica de Madrid. Ciudad Universitaria, 28040 Madrid, Spain.

IEEE Log Number 9407284.

While the propagation speeds of such modes are naturally defined and computed, being strictly related to the eigenvalues of the wave equation for the coupled line structure, in the literature the concept of *modal characteristic impedance* and *modal characteristic impedance matrix* (MZC) has been the subject of different interpretations. The MZC should not be confused with the *characteristic impedance matrix* $\mathbf{Z}_c$, which is a full matrix associated to propagation of voltage and current waves along the coupled lines, over which there is a general agreement and an unambiguous definition. In [12] a full MZC has been first introduced (say $\mathbf{Z}_m$), and it has been used by several other authors, e.g. in [5], [13], [14]. In such matrix, columns are associated to modes and rows to lines. If one loads the lines with the impedances in column k, one gets a perfect match for mode k.

An alternative definition can be found in the literature (see e.g. [10]), according to which the MZC is a diagonal matrix whose elements are the characteristic impedances of the normal modes. This second definition is more strictly related to the decoupled line formulation of the coupled transmission lines problem, since each mode is associated with a characteristic impedance in addition to the proper speed of propagation. This second definition of MZC is dealt with in this paper. For the sake of clarity, throughout this paper we shall refer to such MZC (diagonal) as matrix $\mathbf{Z}_d$. Unlike the former MZC, in the literature matrix $\mathbf{Z}_d$ is computed according to different schemes. It seems that the main issue over which some disagreement appeared is the normalization of the voltage and current eigenvector matrix. Since matrix $\mathbf{Z}_d$ depends on such normalizations, some interesting questions arise quite naturally:

- What is the physical meaning of the different possible normalizations of the current and voltage eigenvector matrices when computing matrix $\mathbf{Z}_d$?
- Is there a unique definition of matrix $\mathbf{Z}_d$?

Because of its arbitrariness in a circuit description of the coupled-lines system, in general it has been paid little attention to the problem of normalization. In [6], [10], [15]–[17] a normalization is assumed, but not justified; in [3] matrix $\mathbf{Z}_d$ is identified with the diagonal matrix of eigenvalues of matrix $\mathbf{Z}_c$; in [18] the eigenvalues of the capacitance p.u.l. matrix are used as modal capacitances p.u.l. (from them one can derive the modal characterist impedances); in [4] an almost diagonal matrix is introduced and the off diagonal elements are then neglected to form a diagonal matrix.

A quite important issue in this context is the relationship between the current eigenvector matrix $\mathbf{M}_i$ and the voltage eigenvector matrix $\mathbf{M}_v$. As pointed out in [2], in general, a normalization for which $\mathbf{M}_i = \mathbf{M}_v$ does not exist. This fact was observed and emphasized also in [19] and [20]. On the other hand, since $\mathbf{M}_i$ and $\mathbf{M}_v$ are generated independently (by two related, but different, eigenvalue problems), one actually finds $2N$ degrees of freedom (N being the size of either matrix).

The purpose of this paper is to give a contribution to the understanding of the different possible normalizations of the current and voltage eigenvector matrices, their implications on matrix $\mathbf{Z}_d$ and their physical meaning. The topics of definition and computation of matrix $\mathbf{Z}_d$ and its relationship with matrix $\mathbf{Z}_c$ are thus dealt with in detail. The main results of this work is the observation (and demonstration) that the numerical value of matrix $\mathbf{Z}_d$ is *dependent on the definition*. The three classical cases mutuated from hybrid-mode analysis are analyzed: the power-current definition, the power-voltage definition and the current-voltage definition. These three different definitions, although applied to a quasi-TEM structure, lead in general to three different values of modal characteristic impedance matrix. Thus, there exist a set of normalizations with a well defined physical meaning.

The work assumes quasi-TEM propagation in lossless, coupled transmission lines in a linear, isotropic, inhomogeneous medium and is organized as follows: after a section which summarizes the main results on the subject, the normalization problem is treated in sections III-IV-V. Section VI is a collection of some numerical examples on two and three coupled microstrip structures in order to provide further evidence to the theoretical results obtained in sections III-IV-V. Although the very important problem of lossy lines is not treated, this contribution should help understand the physical nature of normal modes in quasi-TEM coupled lines and, possibly, answer to the questions raised in this introduction.

II. A Recall on Theory

Our starting point are the well-known time-harmonic generalized telegrapher's equations [7]:

$$\frac{\partial \mathbf{v}'}{\partial z} = -j\omega \mathbf{L}\mathbf{i}' \tag{1}$$

$$\frac{\partial \mathbf{i}'}{\partial z} = -j\omega \mathbf{C}\mathbf{v}'. \tag{2}$$

where $\mathbf{L}$ and $\mathbf{C}$ are respectively the $N \times N$ inductance and capacitance matrices p.u.l. of the system of N conductors and $\mathbf{v}'$, $\mathbf{i}'$ are respectively the voltages and currents along the coupled lines organized as column vectors. With the structures assumed in this work, $\mathbf{L}$ and $\mathbf{C}$ are symmetric positive definite matrices (for a discussion on the properties of such matrices see [21]). Matrix $\mathbf{L}$ is related to matrix $\mathbf{C}_a$ (the capacitance matrix obtained when all dielectrics are removed) by the well known relationship [6]

$$\mathbf{L}\mathbf{C}_a = \mu_0\epsilon_0\mathbf{I} \tag{3}$$

where $\mathbf{I}$ is the identity matrix. By suppressing time dependence ($e^{j\omega t}$) and looking for solutions whose z-dependence is of the type

$$\mathbf{v}'(z) = \mathbf{v}e^{-j\beta z} \tag{4}$$

$$\mathbf{i}'(z) = \mathbf{i}e^{-j\beta z} \tag{5}$$

one finds by substitution the following eigenvalue problems:

$$\frac{1}{c^2}\mathbf{v} = \mathbf{LC}\mathbf{v} \tag{6}$$

$$\frac{1}{c^2}\mathbf{i} = \mathbf{CL}\mathbf{i}. \tag{7}$$

where c is the speed of propagation (unknown). Let now λ_k^2 be the eigenvalues of matrices $\mathbf{LC}$ and $\mathbf{CL}$, then

$$\mathbf{\Lambda} = [\lambda_k] \tag{8}$$

and finally let the eigenvectors of matrices $\mathbf{LC}$ and $\mathbf{CL}$ be organized as columns of matrices $\mathbf{M}_v$ and $\mathbf{M}_i$ respectively. At present no specification is made about their normalization.

By inserting $\mathbf{M}_i$ into (1) one finds the matrix $\overline{\mathbf{M}}_v$ of voltages associated to the eigenvector matrix $\mathbf{M}_i$. It is found as

$$\overline{\mathbf{M}}_v = \mathbf{L}\mathbf{M}_i\mathbf{\Lambda}^{-1} \tag{9}$$

Similarly, by inserting $\mathbf{M}_v$ into (2) one finds the matrix $\overline{\mathbf{M}}_i$ of currents associated to the eigenvector matrix $\mathbf{M}_v$:

$$\overline{\mathbf{M}}_i = \mathbf{C}\mathbf{M}_v\mathbf{\Lambda}^{-1}. \tag{10}$$

It can be shown that

$$\overline{\mathbf{M}}_v = \mathbf{M}_v\mathbf{D}_v \tag{11}$$

and

$$\overline{\mathbf{M}}_i = \mathbf{M}_i\mathbf{D}_i \tag{12}$$

where $\mathbf{D}_v$ and $\mathbf{D}_i$ are diagonal matrices. Since matrices $\mathbf{M}_v$ and $\mathbf{M}_i$ are actually specified with an undetermined right-multiplying diagonal matrix, (11), (12) simply state that by exciting the lines with a voltage eigenvector, a current eigenvector results, and vice-versa. It also means that matrices

$$\mathbf{L}_d = \mathbf{M}_v^{-1}\mathbf{L}\mathbf{M}_i \tag{13}$$

and

$$\mathbf{C}_d = \mathbf{M}_i^{-1}\mathbf{C}\mathbf{M}_v \tag{14}$$

are diagonal. They can define modal inductance and capacitance p.u.l., providing some criterion for normalization is defined.

III. The Definition of Modal Characteristic Impedance Matrix

The diagonal matrices $\mathbf{L}_d$ and $\mathbf{C}_d$ have been usually interpreted as *modal inductance and capacitance p.u.l.*. It is then natural to introduce the concept of *modal characteristic impedance matrix* $\mathbf{Z}_d$, which is related to modal inductance and capacitance [10]. Matrix $\mathbf{Z}_d$, together with the set of eigenvalues λ_k, leads to the well knwon *decoupled formulation* of the coupled transmission lines problem. However, because of the arbitrarity in the definition of both $\mathbf{M}_v$ and $\mathbf{M}_i$, matrices $\mathbf{L}_d$, $\mathbf{C}_d$ and $\mathbf{Z}_d$ are actually undefined.

In a circuit description of the system of lossless coupled transmission lines, the normalization can be chosen arbitrarily, since any normalization adopted is then recovered when one goes back from the modal waves to the line waves description. However, the normalization of the voltage and current eigenvectors can be shown to be related to the physical meaning of matrix $\mathbf{Z}_d$ and establishes a link between the general N-line case, the single-line case and the case of two symmetrical coupled lines, where the even-mode and odd-mode characteristic impedances are unambiguously defined and computed by all authors.

In the following sections it is shown that in the general case, matrix $\mathbf{Z}_d$ takes on values which *depend on the definition*. The three usual cases are considered: power-current, power-voltage and voltage-current. According to the definition one adopts, different values of modal characteristic impedance are found. This is first demonstrated and then verified by applying the three definitions to some two and three-line systems.

A. The Voltage-Current (VI) Case

In this case, the characteristic impedance of each mode takes on a meaning analogous to the ratio V/I for the single-line case. This is the definition of modal characteristic impedance adopted in the literature [10]. The expression is

$$\mathbf{Z}_d^{(\mathrm{VI})} = (\mathbf{M}_v^{-1}\mathbf{C}^{-1}\mathbf{M}_i\mathbf{M}_v^{-1}\mathbf{L}\mathbf{M}_i)^{1/2} \tag{15}$$

which is dependent on the normalization of both $\mathbf{M}_i$ and $\mathbf{M}_v$. For the purpose of this work, we consider matrices $\mathbf{M}_i$ and $\mathbf{M}_v$ normalized according to (18), (24). The associated inductance and capacitance matrices p.u.l. $\mathbf{L}_d^{(\mathrm{VI})}$ and $\mathbf{C}_d^{(\mathrm{VI})}$ are then defined by (13) and (14).

B. The Power-Current (PI) Case

In the PI case, the characteristic impedance $Z_k^{(\mathrm{PI})}$ of each mode assumes the following meaning:

$$\frac{1}{2}\mathbf{v}_k^T\mathbf{i}_k^* = \frac{1}{2}\mathbf{i}_k^T\mathbf{i}_k^* Z_k^{(\mathrm{PI})} \tag{16}$$

where the subscript k indicates the mode and the superscript 'T' indicates transposition. This definition is naturally extended from the single-line case, the difference being that $\mathbf{i}$ is actually a vector in a N-dimensional euclidean space. According to this definition, the MZC is related to power by a measure of the total current associated to the mode.

By indicating with $\overline{\mathbf{m}}_k^v$ the k-th column of matrix $\overline{\mathbf{M}}_v$ and with $\mathbf{m}_k^i$ the k-th column of $\mathbf{M}_i$ one finds

$$Z_k^{(\mathrm{PI})} = \frac{(\overline{\mathbf{m}}_k^v)^T(\mathbf{m}_k^i)^*}{(\mathbf{m}_k^i)^T(\mathbf{m}_k^i)^*} \tag{17}$$

which is *independent of the normalization of matrix* $\mathbf{M}_i$ (matrix $\mathbf{M}_v$ does not appear). However, by normalizing $\mathbf{M}_i$ so that

$$(\mathbf{m}_k^i)^T(\mathbf{m}_k^i)^* = 1 \qquad k = 1, 2 \ldots N \tag{18}$$

the diagonal matrix of PI modal characteristic impedances $\mathbf{Z}_d^{(\mathrm{PI})}$ takes on the following form:

$$\mathbf{Z}_d^{(\mathrm{PI})} = \mathbf{\Lambda}^{-1}\mathbf{M}_i^T\mathbf{L}\mathbf{M}_i^* \tag{19}$$

where we have used (9), and the associated inductance and capacitance matrices are

$$\mathbf{L}_d^{(\mathrm{PI})} = \mathbf{\Lambda}\mathbf{Z}_d^{(\mathrm{PI})} \tag{20}$$

$$\mathbf{C}_d^{(\mathrm{PI})} = \mathbf{\Lambda}(\mathbf{Z}_d^{(\mathrm{PI})})^{-1} \tag{21}$$

The Power-Voltage (PV) Case

A second possible definition relates the characteristic impedance $Z_k^{(\mathrm{PV})}$ of each mode to power through

$$\frac{1}{2}\mathbf{i}_k^T\mathbf{v}_k^* = \frac{1}{2}\mathbf{v}_k^T\mathbf{v}_k^*(Z_k^{(\mathrm{PV})})^{-1}. \tag{22}$$

In this case

$$(Z_k^{(\mathrm{PV})})^{-1} = \frac{(\overline{\mathbf{m}}_k^i)^T(\mathbf{m}_k^v)^*}{(\mathbf{m}_k^v)^T(\mathbf{m}_k^v)^*} \tag{23}$$

which is *independent of the normalization of matrix* $\mathbf{M}_v$ (matrix $\mathbf{M}_i$ does not appear). If matrix $\mathbf{M}_v$ is normalized so that

$$(\mathbf{m}_k^v)^T(\mathbf{m}_k^v)^* = 1 \qquad k = 1, 2 \ldots N \tag{24}$$

the diagonal matrix of PV modal characteristic impedances takes on the form

$$\mathbf{Z}_d^{(\mathrm{PV})} = (\mathbf{\Lambda}^{-1}\mathbf{M}_v^T\mathbf{C}\mathbf{M}_v^*)^{-1} \tag{25}$$

having used (10), and the inductance and capacitance matrices p.u.l. are derived according to (20), (21).

IV. Comparison Between the Three Definitions

It is shown in this section that the three formulations lead to different values of modal impedances. In order to make such comparison we first normalize matrices $\mathbf{M}_i$ and $\mathbf{M}_v$ so that (18), (24) hold. This is done in order to use the matrix expression for mode impedance. Note that any normalization can be used, since matrices $\mathbf{Z}_d^{(\mathrm{PI})}$ and $\mathbf{Z}_d^{(\mathrm{PV})}$ are normalization independent. The three results obtained with the three definitions are repeated here for convenience:

$$\mathbf{Z}_k^{(\mathrm{PI})} = \mathbf{\Lambda}^{-1}\mathbf{M}_i^T\mathbf{L}\mathbf{M}_i, \tag{26}$$

$$\mathbf{Z}_d^{(\mathrm{PV})} = (\mathbf{\Lambda}^{-1}\mathbf{M}_v^T\mathbf{C}\mathbf{M}_v)^{-1}, \tag{27}$$

$$\mathbf{Z}_d^{(\mathrm{VI})} = (\mathbf{M}_v^{-1}\mathbf{C}^{-1}\mathbf{M}_i\mathbf{M}_v^{-1}\mathbf{L}\mathbf{M}_i)^{1/2}. \tag{28}$$

and the asterisk has been dropped since matrices $\mathbf{M}_\imath$ and $\mathbf{M}_v$ can be chosen real [7]. Note also that because of $\mathbf{Z}_d^{(PI),(PV)}$ being diagonal, matrix $\mathbf{\Lambda}^{-1}$ can be moved to the right.

By inspection on the three formulas above, one can easily find that the three definitions lead to the same diagonal matrix of modal characteristic impedance only if

$$\mathbf{M}_\imath^T \mathbf{M}_v = \mathbf{I} \tag{29}$$

where $\mathbf{I}$ is the identity matrix. It is pointed out that the term in the left side in (29) is always a diagonal matrix, but since the two matrices $\mathbf{M}_\imath$ and $\mathbf{M}_v$ are completely specified (their normalization has been defined), (29) is in effect a further condition on the *normalized* eigenvector matrices. Bacause of Cauchy's inequality (the equal sign applies), (29) together with (18), (24) implies

$$\mathbf{M}_\imath = \mathbf{M}_v \mathbf{D} \tag{30}$$

$\mathbf{D}$ being a diagonal matrix. By substituting (30) into (29), because of (24) one gets that (30) implies

$$\mathbf{M}_v^T \mathbf{M}_v = \mathbf{I} \tag{31}$$

i.e. matrix $\mathbf{M}_v$ must be orthogonal. This is the condition which must be fulfilled if the three definitions of modal impedance are to coincide. The implication of the orthogonality of matrix $\mathbf{M}_v$ (and therefore $\mathbf{M}_i$), is here reported for clarity, but it can be found in [19]. By substituting (31) in (6), (7), one finds:

$$\mathbf{LC} = (\mathbf{LC})^T \tag{32}$$

which can be written as

$$\mathbf{LC} = \mathbf{CL}. \tag{33}$$

We have thus established that the condition for which the three definitions of modal characteristic impedance coincide is equivalent to say that matrices $\mathbf{L}$ and $\mathbf{C}$ commute. This is the case, for instance, when one analyzes two symmetrical lines in an inhomogenous medium, but, in general, (33) is not valid, and the three definitions lead to three different matrices $\mathbf{Z}_d$. The condition established by (33) is a rather strong one. If it is verified, the eigenvalues of matrices $\mathbf{L}$ and $\mathbf{C}$ and the eigenvectors $\mathbf{M}$ of either of them are sufficient to describe the problem since matrices $\mathbf{M}^T\mathbf{CM}$, $\mathbf{M}^T\mathbf{LM}$, $\mathbf{M}^T\mathbf{LCM}$ and $\mathbf{M}^T\mathbf{CLM}$ are all diagonal, and the eigenvalues of both $\mathbf{LC}$ and $\mathbf{CL}$ are the product of the eigenvalues of $\mathbf{L}$ and $\mathbf{C}$.

V. The Characteristic Impedance Matrix

In the literature, many authors introduce the characteristic impedance matrix $\mathbf{Z}_c$. It is usually defined through

$$\overline{\mathbf{M}}_v = \mathbf{Z}_c \mathbf{M}_i \tag{34}$$

and is thus a full matrix linking in some way travelling voltages and currents along the coupled line system [8]. By substituting (9) in (34) one finds

$$\mathbf{Z}_c = \mathbf{L}\mathbf{M}_\imath \mathbf{\Lambda}^{-1} \mathbf{M}_i^{-1}. \tag{35}$$

An alternative, but equivalent, way to compute $\mathbf{Z}_c$ is found by exciting the lines with $\mathbf{M}_v$, i.e.

$$\overline{\mathbf{M}}_\imath = (\mathbf{Z}_c)^{-1}\mathbf{M}_v \tag{36}$$

to find

$$\mathbf{Z}_c = \mathbf{M}_v \mathbf{\Lambda} \mathbf{M}_v^{-1} \mathbf{C}^{-1}. \tag{37}$$

It can be shown that the two expression lead to the same matrix $\mathbf{Z}_c$.

Matrix $\mathbf{Z}_c$ is normalization independent. Thus, one can apply (18), (24) and easily verify that the following equations hold:

$$\mathbf{Z}_d^{(\mathrm{PI})} = \mathbf{M}_i^T \mathbf{Z}_c \mathbf{M}_\imath \tag{38}$$

$$\mathbf{Z}_d^{(\mathrm{PV})} = \mathbf{M}_v^{-1} \mathbf{Z}_c (\mathbf{M}_v^T)^{-1} \tag{39}$$

$$\mathbf{Z}_d^{(\mathrm{VI})} = \mathbf{M}_v^{-1} \mathbf{Z}_c \mathbf{M}_\imath. \tag{40}$$

The three previous equations have the following meaning: there are three possible diagonalizations of matrix $\mathbf{Z}_c$, and either of them can be used as a definition of modal characteristic impedance. However, they have a specific physical meaning, which has been clearly shown in the previous sections. The basis for diagonalization are obtained from the two eigenvalue problems relative to matrices $\mathbf{LC}$ and $\mathbf{CL}$ through suitable normalization.

Equations (38)–(40) can be easily manipulated to express the mode impedances in a different and interesting way: by using (35) to express $\mathbf{Z}_c$ and (14) one finds

$$\mathbf{Z}_d^{(\mathrm{PI})} = \mathbf{M}_i^T \mathbf{M}_v \mathbf{\Lambda} \mathbf{C}_d^{-1} \tag{41}$$

while by using (37) and a few manipulations

$$\mathbf{Z}_d^{(\mathrm{PV})} = \mathbf{\Lambda} \mathbf{C}_d^{-1} (\mathbf{M}_i^T \mathbf{M}_v)^{-1} \tag{42}$$

and finally

$$\mathbf{Z}_d^{(\mathrm{VI})} = (\mathbf{Z}_d^{(\mathrm{PI})} \mathbf{Z}_d^{(\mathrm{PV})})^{1/2}. \tag{43}$$

Thus, by using the VI definition one finds the geometrical mean between the value found with the PI and that found with the PV definitions, in strict analogy with the single-line case. Note that matrix $\mathbf{Z}_d^{(\mathrm{VI})}$ can be generated according to (43) by two *normalization-independent* matrices. Equations (41), (42) also enlight the key role of the diagonal matrix $\mathbf{M}_i^T \mathbf{M}_v$ in the computation of modal impedances (it is recalled that we always deal with normalized matrices). It acts as a switch to the desired definition of modal impedance. It also yields a measure of the difference between the values found according to the three definitions, depending on how different matrix $\mathbf{M}_i^T \mathbf{M}_v$ is from the identity matrix. This last information is then *separated for each mode*, which means that a "1" in position k of $\mathbf{M}_i^T \mathbf{M}_v$ indicates that the three definitions lead to the same value of modal impedance *for mode k*.

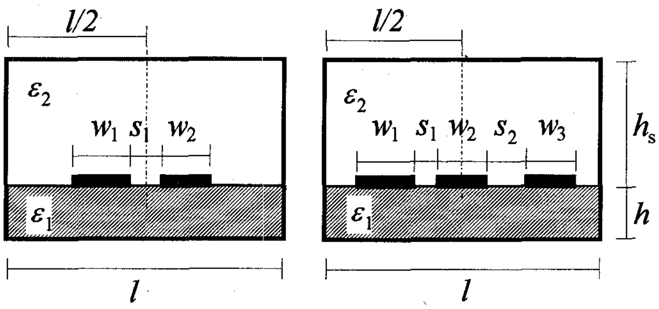


Fig. 1. Physical parameters of a two and three-line system.

$$\mathbf{C}/\epsilon_0 = \begin{bmatrix} 54.604 & -6.634 \\ -6.634 & 22.704 \end{bmatrix} \quad \mathbf{L}/\mu_0 = \begin{bmatrix} 0.13059 & 0.04963 \\ 0.04963 & 0.2846 \end{bmatrix}$$

$$\mathbf{M}_v = \begin{bmatrix} 0.74065 & -0.2762 \\ 0.67189 & 0.9611 \end{bmatrix} \quad \mathbf{M}_i = \begin{bmatrix} 0.9611 & -0.67189 \\ 0.2762 & 0.74065 \end{bmatrix}$$

$$\mathbf{\Lambda}\cdot 10^9 = \begin{bmatrix} 8.849 & 0 \\ 0 & 8.0988 \end{bmatrix} \quad \mathbf{Z}_c(\Omega) = \begin{bmatrix} 18.75 & 6.33 \\ 6.33 & 42.91 \end{bmatrix}$$

Fig. 2. Matrices relative to two unsymmetrical coupled microstrips. $w_1/l = 0.4$, $w_2/l = 0.1$, $s/l = 0.02$, $h/l = 0.1$, $h_s/l = 0.4$, $\epsilon_1 = 10\epsilon_0$, $\epsilon_2 = \epsilon_0$ (see Fig. 1).

TABLE I
CHARACTERISTIC IMPEDANCE (Ω) FOR A STRUCTURE OF TWO COUPLED MICROSTRIPS. $w_1/l = 0.4$, $w_2/l = 0.1$, $s/l = 0.02$, $h/l = 0.1$, $h_s/l = 0.4$, $\epsilon_1 = 10\epsilon_0$, $\epsilon_2 = \epsilon_0$ (SEE FIG. 1).

	c-mode	π-mode
P-I	25.70	23.95
V-I	28.64	26.69
P-V	31.91	29.74

TABLE II
CHARACTERISTIC IMPEDANCE (Ω) FOR A STRUCTURE OF THREE COUPLED MICROSTRIPS: $w_1/l = 0.1$, $w_2/l = 0.1$, $w_3/l = 0.1$, $s_1/l = 0.02$, $s_2/l = 0.02$, $h/l = 0.1$, $h_s/l = 0.3$, $\epsilon_1 = 10\epsilon_0$, $\epsilon_2 = \epsilon_0$ (SEE FIG. 1). THE SIGNS OF THE VOLTAGE EIGENVECTORS ARE SHOWN

	mode 1 $+-+$	mode 2 $+0-$	mode 3 $+++$
P-I	26.65	40.01	67.17
V-I	26.69	40.01	67.27
P-V	26.73	40.01	67.37

TABLE III
CHARACTERISTIC IMPEDANCE (Ω) FOR A STRUCTURE OF THREE COUPLED MICROSTRIPS. $w_1/l = 0.2$, $w_2/l = 0.05$, $w_3/l = 0.02$, $s_1/l = 0.01$, $s_2/l = 0.01$, $h/l = 0.1$, $h_s/l = 0.3$, $\epsilon_1 = 10\epsilon_0$, $\epsilon_2 = \epsilon_0$ (SEE FIG. 1). THE SIGNS OF THE VOLTAGE EIGENVECTORS ARE SHOWN

	mode 1 $+-+$	mode 2 $+--$	mode 3 $+++$
P-I	30.54	40.13	47.16
V-I	31.78	50.20	57.48
P-V	33.07	62.79	70.05

VI. RESULTS

Some numerical simulations have been performed to test the various definitions of modal characteristic impedance. At first a pair of shielded coupled microstrip lines of different widths have been analyzed. Such lines support a π-mode (voltages of the same sign) and a c-mode (voltages of opposite sign) [1]. The partial results on the matrices defined in the paper are shown in Fig. 2 (the physical parameters are defined in Fig. 1).

The results on modal characteristic impedance using the three definitions above introduced are given in Table I. Note that there is a considerable difference in the three values for each mode. As pointed out in the previous section, the VI definition leads to values which are the geometrical mean between the other two, while the PI definition leads to the lowest values for each mode and the PV definition to the highest.

A second example is shown in Table II. In this case a structure composed of 3 symmetrical coupled microstrips has been analyzed. It is interesting to observe that although the structure is symmetrical the three definitions of $\mathbf{Z}_d$ lead to slighlty different results. There is however one mode for which the three definitions coincide (mode 2). The mode eigenvector is the following: 0.707, 0, −0.707. Thus the mode is completely antisymmetrical and has equal voltages (in magnitude) on the two outer lines. No difference results in its modal impedance from the application of the three definitions. A '1' appears in the corresponding position of in matrix $\mathbf{M}_i^T\mathbf{M}_v$. This is actually the odd mode of the *pair* of external lines when the center line is connected to ground. Since such system of two lines is symmetrical, no difference results from the application of the three definitions.

In Table III a third example is shown. In this case a strongly asymmetrical structure composed of three coupled microstrip lines has been analyzed. The difference in the values of the PI, PV and VI modal characteristic impedances is very strong. The mode patterns (signs) of the voltage eigenvectors are also shown in the table.

The different values of characteristic impedance in general structures can be justified from a rather intuitive point of view. One should bear in mind that modal characteristic impedance is related to a mode propagating along the coupled lines. Each mode has an associated voltage pattern and current pattern, thus, although power is well defined, voltage and current are actually functions of position on the transmission line cross-section (the transmission line associated to each mode comprises all lines). Thus if one introduces some measure of such functions, in order to define characteristic impedance, it is not surprising to observe a dependance on the measure of function "v" and function "i." Actually, the same value of characteristic impedance is found only when the two functions are the same ($\mathbf{M}_i = \mathbf{M}_v$). This happens, for instance, in two symmetrical coupled lines. The even and odd mode current and voltage patterns are respectively symmetrical and antisymmetrical and the voltage and current measures can be defined simply by taking the voltage and current along either of the lines.

A further point concerns full-wave analysis of multiconductor lines. It is well known that the PI and PV definitions of modal characteristic impedance are commonly used in the hybrid-mode analysis of single and coupled transmission lines structures. When comparing the results obtained by the hybrid-mode analysis at low frequencies with the "static" case (quasi-TEM) one should use congruent definitions in both cases.

As a final comment it is pointed out that in all the tables shown, the values of matrices **C** and **L** (actually $\mathbf{C}_a$) were computed by a numerical method [23], and they are affected by the number of basis functions used to discretize the charge density on the strips. The numbers shown are thus approximations to the exact values. The number of basis functions used in the computations is believed to yield about three-digits accuracy.

VII. Conclusions

The problem of defining and computing modal characteristic impedance in coupled quasi-TEM transmission lines in a inhomogenous medium has been comprehensively dealt with. It is found that the three usual definitions of characteristic impedance (power-current, power-voltage and voltage-current), although they are applied to a quasi-TEM structure, lead in general to different values of modal characteristic impedances. An exception to this occurs when matrices **L** and **C** commute. In that case the three definitions lead to the same values of modal characteristic impedances.

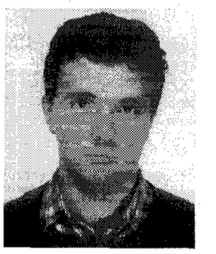
The theoretical results have been then confirmed by some numerical examples on systems of two and three coupled transmission lines.

Acknowledgment

This paper is based on work performed during a stay of the first author at SSR Department of the Politechnic University of Madrid. The author wishes to thank prof. Felix Perez-Martinez for making possible this cooperation.

References

[1] V. K. Tripathi, "Equivalent circuits and characteristics of inhomogenous nonsymmetrical coupled-line two-port circuits," *IEEE Trans. Microwave Theory Tech.*, Feb. 1977, pp. 140–142.
[2] V. K. Tripathi, "On the analysis of symmetrical three-line microstrip circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. 25, Sept. 1977, pp. 726–729.
[3] G. Ghione, I. Maio, and G. Vecchi, "Modeling of multiconductor buses and analysis of crosstalk, propagation delay and pulse distortion in high-speed GaAs logic circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. 37, Mar. 1989, pp. 445–456.
[4] H. You, M. Soma, "Crosstalk and transient analyses of high-speed Interconnects and Packages," *IEEE J. Solid-State Circuits*, vol. 26, Mar. 1991, pp. 319–329.
[5] J. E. Schutt-Aine, R. Mittra, "Analysis of pulse propagation in coupled transmission lines," *IEEE Trans. Circuits and Syst.*, Vol. 32, Dec. 1985, pp. 1214–1219.
[6] F. Y. Chang, "Transient analysis of lossless coupled transmission lines in a inhomogeneous dielectric," *IEEE Trans. Microwave Theory Tech.*, vol. 18, pp. 616–626, Sept. 1970.
[7] K. Marx, "Propagation modes, equivalent circuits and characteristic terminations for multiconductor transmission lines with inhomogeneous dielectric," *IEEE Trans. Microwave Theory Tech.*, vol. 21, pp. 450–457, July 1973.
[8] A. R. Djordjević, T. Sarkar, and R. F. Harrington, "Time-domain response of multiconductor transmission lines," *Proc. IEEE*, vol. 75, no. 6, pp. 743–764, June 1987.
[9] A. R. Djordjević, T. Sarkar, "Analysis of time response of lossy multiconductor transmission lines networks," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 898–908, Oct. 1987.
[10] F. Romeo, M. Santomauro, "Time-domain simulation of n coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 35, pp. 131–137, Feb. 1987, .
[11] T. Dhaene, D. De Zutter, "CAD-oriented general circuit description of uniform coupled lossy dispersive waveguide structures," *IEEE Trans. Microwave Theory Tech.*, Vol. 40, July 1992, pp. 1545–1554.
[12] N. Nagai, R. Hatori, "Equivalent circuit representation of coupled multiwire line and its application," *Monograph Series, Res. Inst. Appl. Electricity*, Sapporo, Japan, no. 24, pp. 7–30, 1977.
[13] J. E. Schutt-Aine, R. Mittra, "Nonlinear transient analysis of coupled transmission lines," *IEEE Trans. Circuits and Syst.*, vol. 36, No. 7, pp. 959–967, July 1989.
[14] F. L. Mesa, G. Cano, F. Medina, R. Marques, and M. Horno, "On the quasi-TEM and full-wave approaches applied to coplanar multistrip on lossy dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-40, pp. 524–531, Mar. 1992.
[15] V. Dvorak, "Numerical solution of the transient response of a distributed parameter transformer," *IEEE Trans. Circuit Theory*, pp. 270–273, May, 1970.
[16] J. Alonso, J. Borja, F. Perez, "An efficient spice model for lossy and dispersive coupled transmission lines," *IEEE Symp. Circ. Syst.*, pp. 2713–2716, 1991.
[17] N. Orhanovic, V. K. Tripathi, "Nonlinear transient analysis of coupled RLGC lines by the method of characteristics," *Int. J. Microwave, Millimeter-Wave CAD*, vol. 2, no. 2, pp. 108–115, 1992.
[18] C. R. Paul, "A simple SPICE model for coupled transmission lines," *IEEE MTT-Symp.*, pp. 327–333, 1988.
[19] P. Crozat, A. Zounon, and R. Adde, "Condition of modal analysis in time domain of lossy coupled lines," *Electron. Lett.*, vol. 24, no. 20, pp. 1289–1290, Sept. 1988.
[20] C. Nguyen, "On the analysis of parallel coupled transmission lines in a inhomogenous medium," *Microwave and Optical Tech. Lett.*, vol. 3, no. 9, pp. 308–310, Sept. 1990.
[21] C. W. Ho, "Theory and computer-aided analysis of lossless transmission lines," *IBM J. Res. Development*, pp. 249–255, May 1973.
[22] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.
[23] G. G. Gentili, G. Macchiarella, "A quasi-static mixed spectral-space domain analysis of shielded stripline-like transmission lines with finite metalization thickness," *IEEE Trans. Microwave Theory Tech.*, pp. 249–255, Feb. 1994.



Gian Guido Gentili was born in Turin, Italy. He received the "laurea" degree in electronic engineering from Politecnico di Milano, Milano, Italy, in 1987. From 1987 to 1989 he worked at Politecnico di Milano as a scholarship holder, developing CAD tools for the electromagnetic analysis of microstrip circuits.

In 1989, as a researcher, he joined the "Centro Studi sulle Telecomunicazioni Spaziali" (CSTS-CNR), which resides at Politecnico di Milano. Since then he has worked on the application of numerical methods for the modelling and design of microwave and millimeter wave passive structures.

Magdalena Salazar-Palma (M'89) was born in Granada, Spain. She received her degrees in Telecommunication Engineering from the Polytechnic University of Madrid, where she is an associate professor of the Signals, Systems and Radiocommunications Department at the Technical School of Telecommunication Engineering.

Her interests are focused on electromagnetic field theory, numerical methods for microwave structures and passive components analysis, as well as on hybrid and monolithic microwave integrated circuits.